

Problem 2.16

[Difficulty: 2]

2.16 A velocity field is given by $\vec{V} = ayt\hat{i} - bx\hat{j}$, where $a = 1 \text{ s}^{-2}$ and $b = 4 \text{ s}^{-1}$. Find the equation of the streamlines at any time t . Plot several streamlines at $t = 0 \text{ s}$, $t = 1 \text{ s}$, and $t = 20 \text{ s}$.

Given: Velocity field

Find: Equation of streamlines; Plot streamlines

Solution:

Streamlines are given by $\frac{v}{u} = \frac{dy}{dx} = \frac{-b \cdot x}{a \cdot y \cdot t}$

So, separating variables $a \cdot t \cdot y \cdot dy = -b \cdot x \cdot dx$

Integrating $\frac{1}{2} \cdot a \cdot t \cdot y^2 = -\frac{1}{2} \cdot b \cdot x^2 + C$

The solution is $y = \sqrt{C - \frac{b \cdot x^2}{a \cdot t}}$

For $t = 0 \text{ s}$ $x = c$ For $t = 1 \text{ s}$ $y = \sqrt{C - 4x^2}$ For $t = 20 \text{ s}$ $y = \sqrt{C - \frac{x^2}{5}}$

$t = 0$

C = 1 C = 2 C = 3

x	y	y	y
0.00	1.00	2.00	3.00
0.10	1.00	2.00	3.00
0.20	1.00	2.00	3.00
0.30	1.00	2.00	3.00
0.40	1.00	2.00	3.00
0.50	1.00	2.00	3.00
0.60	1.00	2.00	3.00
0.70	1.00	2.00	3.00
0.80	1.00	2.00	3.00
0.90	1.00	2.00	3.00
1.00	1.00	2.00	3.00
1.10	1.00	2.00	3.00
1.20	1.00	2.00	3.00
1.30	1.00	2.00	3.00
1.40	1.00	2.00	3.00
1.50	1.00	2.00	3.00
1.60	1.00	2.00	3.00
1.70	1.00	2.00	3.00
1.80	1.00	2.00	3.00
1.90	1.00	2.00	3.00
2.00	1.00	2.00	3.00

$t = 1 \text{ s}$

C = 1 C = 2 C = 3

x	y	y	y
0.000	1.00	1.41	1.73
0.025	1.00	1.41	1.73
0.050	0.99	1.41	1.73
0.075	0.99	1.41	1.73
0.100	0.98	1.40	1.72
0.125	0.97	1.39	1.71
0.150	0.95	1.38	1.71
0.175	0.94	1.37	1.70
0.200	0.92	1.36	1.69
0.225	0.89	1.34	1.67
0.250	0.87	1.32	1.66
0.275	0.84	1.30	1.64
0.300	0.80	1.28	1.62
0.325	0.76	1.26	1.61
0.350	0.71	1.23	1.58
0.375	0.66	1.20	1.56
0.400	0.60	1.17	1.54
0.425	0.53	1.13	1.51
0.450	0.44	1.09	1.48
0.475	0.31	1.05	1.45
0.500	0.00	1.00	1.41

$t = 20 \text{ s}$

C = 1 C = 2 C = 3

x	y	y	y
0.00	1.00	1.41	1.73
0.10	1.00	1.41	1.73
0.20	1.00	1.41	1.73
0.30	0.99	1.41	1.73
0.40	0.98	1.40	1.72
0.50	0.97	1.40	1.72
0.60	0.96	1.39	1.71
0.70	0.95	1.38	1.70
0.80	0.93	1.37	1.69
0.90	0.92	1.36	1.68
1.00	0.89	1.34	1.67
1.10	0.87	1.33	1.66
1.20	0.84	1.31	1.65
1.30	0.81	1.29	1.63
1.40	0.78	1.27	1.61
1.50	0.74	1.24	1.60
1.60	0.70	1.22	1.58
1.70	0.65	1.19	1.56
1.80	0.59	1.16	1.53
1.90	0.53	1.13	1.51
2.00	0.45	1.10	1.48

